

MATH 2E Prep: Level Sets and Surfaces

Facts to Know:

1. Level sets of 2-variable functions $f(x, y)$ are:

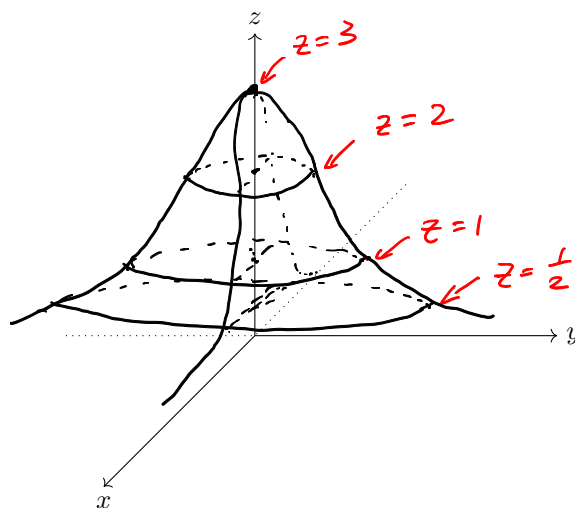
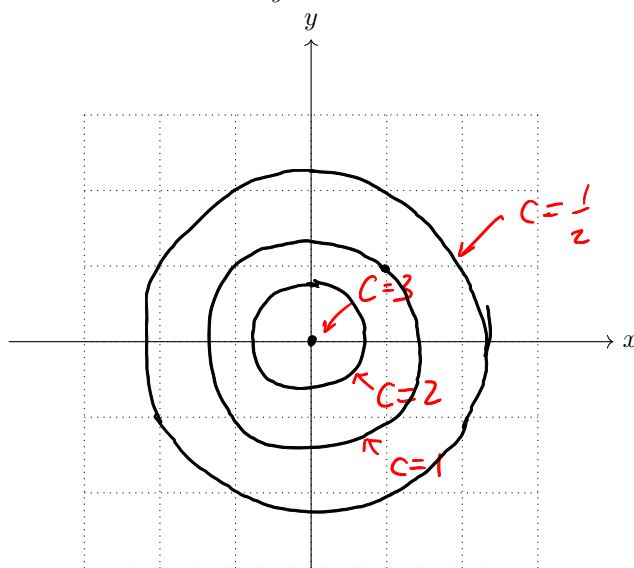
- Defined by $f(x, y) = C$,
- Curves on the xy -plane, (sometimes pts or empty)
- On the graph of $z = f(x, y)$ in $\mathbb{R}^3$. (graph of $f(x)$ on $\mathbb{R}^2$ is $y = f(x)$)

2. Level sets of 3-variable functions $g(x, y, z)$ are:

- Defined by $g(x, y, z) = C$,
- Surfaces in $\mathbb{R}^3$.

Examples:

1. Sketch the level sets $\frac{3}{x^2 + y^2 + 1} = C$ for $C = 3, 2, 1, \frac{1}{2}$ on xy -plane, then use the level sets to sketch the graph of $z = \frac{3}{x^2 + y^2 + 1}$ in $\mathbb{R}^3$



$$C=3, \quad \frac{3}{x^2 + y^2 + 1} = 3, \quad \Rightarrow x^2 + y^2 = 0, \quad \text{pt.}$$

$$C=2, \quad \frac{3}{x^2 + y^2 + 1} = 2 \Rightarrow x^2 + y^2 = \frac{1}{2}$$

$$C=1, \quad x^2 + y^2 = 2$$

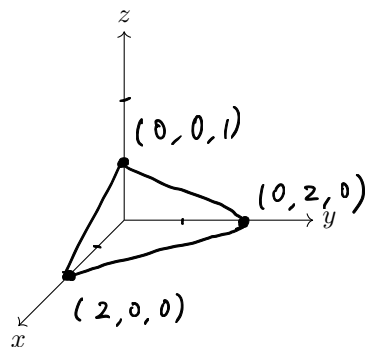
$$C=\frac{1}{2}, \quad x^2 + y^2 = 5$$

2. Sketch the following level sets in $\mathbb{R}^3$:

- The Plane $x + y + 2z = 2$

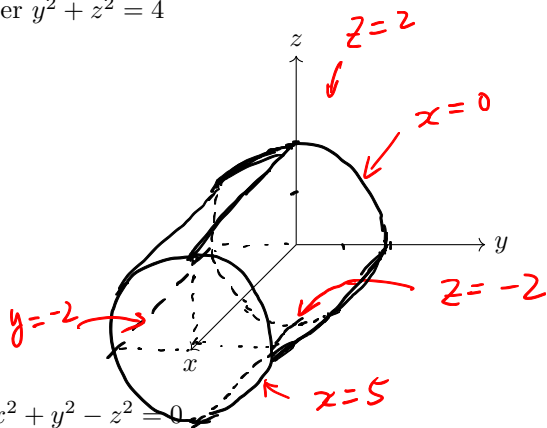
$$f(x, y, z)$$

Graph:



- The Cylinder $y^2 + z^2 = 4$

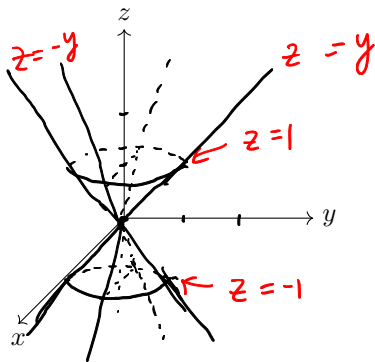
Graph:



- The Cone $x^2 + y^2 - z^2 = 0$

$$x^2 + y^2 = z^2$$

Graph:



- The Sphere $x^2 + y^2 + z^2 = 1$

Graph:

